MATERIALS USED

The n-butane was research grade with a purity of over 99.9%. The nitrogen used was research grade and had a purity of over 99.9%.

RESULTS

The analyses of the equilibrium mixtures are shown in Table 1 and Figure 1. In Figure 2 is shown the equilibrium distribution ratios, $K = y/x$, which were calculated from the smoothed data from Figure 1.

In Figure 1 there are also shown the interpolated data of Akers, et al. (*1*) on the same system. The considerable disagreement which results between this work and that of Akers, et al., at temperatures above 100°F., may be attributed in large part to the fact that their cell did not have a glass window for observation of the phases and also to the unusual sensitivity required in the use of an Edwards gas density balance (their method of analysis) for precision data in the binary system such as nitrogen and n-butane. Since the limiting working pressure of our equilibrium cell is 3,500 lb./sq. in., this investigation did not cover the critical region of the system at 100°F.

ACCURACY

The accuracy of the indicated pressure is ± 4 lb./sq. in. abs. Other sources of error are temperature variations during the recirculation, sampling procedures, pressure gauge readings, and analytical errors. Consideration of the analytical procedure shows that the analyses were reliable to ± 0.002 mole fraction. The maximum temperature variation within the cell during the recirculation was ± 0.5°F. and was ± 0.1°F. during settling and sampling time.

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Use of the Mechanical Energy Balance for Two-Phase Flow

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Attention is called to the improper use of the mechanical-energy balance for describing the distribution of the static pressure drop for two-phase flow systems into components of frictional, head, and momentum terms. Consider an upward gas-liquid flow system for an annular flow regime in a vertical pipe. The proper momentum balance with transfer between phases is given in references *1* and *2*:

$$dP + \frac{1}{g_c A} d[W_G V_G + W_L V_L] + \frac{g}{g_c}\left[\rho_L \frac{A_L}{A} + \rho_G \frac{A_G}{A}\right]dz = \frac{A_L}{A}\left(\frac{dP}{dz}\right)_{LTP} dz + \frac{A_G}{A}\left(\frac{dP}{dz}\right)_{GTP} dz \qquad (1)$$

For a single-phase (turbulent) flow system the mechanical energy balance is written as

$$\frac{dP}{\rho dz} + \frac{VdV}{g_c dz} + \frac{g}{g_c} + \frac{dF}{dz} = 0 \qquad (2)$$

Some investigators have tried to apply Equation (2) or a similar form to a two-phase system by weighting each phase by the mass flow (W_L or W_G) and then adding the equations together. This procedure is wrong. First Equation (2) does not apply for the case of mass transfer between phases. Even if the momentum and friction terms are considered negligible, the procedure is still wrong and yields an improper evaluation of the head term.

Equation (2) may be used for a two-phase flow system with no transfer between phases by recognizing that the energy balance per pound mass of each phase is weighted by the mass of the phase in the differential volume contained in the section dz ($\rho_G A_G dz$ for the gas phase, $\rho_L A_L dz$ for liquid phase). With this weighting, and summing the equations, one obtains Equation (3):

$$dP + \frac{1}{g_c A}\cdot[W_G dV_G + W_L dV_L] + \frac{g}{g_c}\left[\rho_L \frac{A_L}{A} + \rho_G \frac{A_G}{A}\right]dz + \left[\frac{A_L \rho_L}{A} dF_L + \frac{A_G \rho_G}{A} dF_G\right] = 0 \qquad (3)$$

Note that

$$\rho_L \frac{dF_L}{dz} = -\left(\frac{dP}{dz}\right)_{LTP}$$

$$\rho_G \frac{dF_G}{dz} = -\left(\frac{dP}{dz}\right)_{GTP}$$

For constant W_G and W_L Equation (1) reduces to Equation (3).

Perhaps the easiest way to demonstrate that W_L and W_G are the wrong weighting factors is to show that in dt time the gas phase moves a distance $W_G dt/\rho_G A_G = V_G dt$, whereas the liquid moves $W_L dt/\rho_L A_L = V_L dt$. The only way for the incremental distance dz to be the same for the gas and liquid phase is for $V_G = V_L$, which is a very special case.

NOTATION

- A = cross-sectional area of flow (sq.ft.), $A = A_G + A_L$
- F = frictional energy loss per unit mass (ft.-lb. force/lb. mass)
- g = gravitational constant, (4.17 x 10^8 ft./hr.^{2})
- g_c = conversion factor (4.17 x 10^8 lb. mass-ft./lb. force-hr.^{2})
- P = pressure (lb. force/sq.ft.)
- t = time (hr.)
- V = velocity (ft./hr.)
- W = mass flow rate, (lb. mass/hr.)
- z = vertical height, (ft.)
- ρ = density (lb. mass/cu.ft.)

Subscripts

- G = gas phase
- GTP = frictional loss in gas in two-phase flow
- L = liquid phase
- LTP = frictional loss in liquid in two-phase flow

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